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Corrigendum

Corrigendum to "Multiple stability and uniqueness of the limit cycle in a Gause-type predator–prey model considering the Allee effect on prey" [Nonlinear Anal. RWA 12 (2011) 2931–2942]

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ABSTRACT

This work deals with some typographical mistakes into the above-referenced paper. Although they do not affect the main results, it is necessary to make due corrections.

We affirm that the results and conclusions obtained are correct and the errors have no further implications in the aforementioned paper.

1. Introduction

The aim of this presentation is to correct several minor typographical mistakes that crept into the article by González-Olivares et al. [1].

Although those typos do not affect the main results, they may, unfortunately, lead to a misunderstanding of the results obtained.

It should be stressed that errors appear in both the statements and proofs of Theorems 4 and 7. For this reason, we provide the corresponding corrections for an adequate understanding of these aspects of the aforementioned theorems.

2. The correction

The statement of Theorem 4 in [1] (page 2935) has typos in the inequalities for S. It must be replaced by the following one:

Theorem 1. *(Theorem 4 in [1]) Let us assume that* $(u^*, v^s) \in W^s(M, 0)$ *and* $(u^*, v^u) \in W^u(1, 0)$*, where* v^s *and* v^u *are functions of the parameters* A, E, S *and* M*. Let us further assume that* $v^s \geq v^u$.

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(a) *If* $A > \frac{-3E^2+2EM+2E-M}{2E-M-1}$, *then the trace is negative and the equilibrium* Q_e *is a local attractor.*

(a1) *If* $S > \frac{\left(-3E^2-2EA+2E+A\right)^2}{4(1-E)(A+E)}$, *then* Q_e *is an attracting focus.*

(a2) *If* $S < \frac{\left(-3E^2-2EA+2E+A\right)^2}{4(1-E)(A+E)}$, *then* Q_e *is an attracting node.*

(b) *If* $A < \frac{-3E^2+2EM+2E-M}{2E-M-1}$, *then the trace is positive and the equilibrium* Q_e *is a repellor.*

(b1) *If* $S > \frac{\left(-3E^2-2EA+2E+A\right)^2}{4(1-E)(A+E)}$, *then* Q_e *is an unstable focus surrounded by a stable limit cycle.*

(b2) *If* $S < \frac{\left(-3E^2-2EA+2E+A\right)^2}{4(1-E)(A+E)}$, *then* Q_e *is an unstable node and the limit cycle disappears. In this last case, the singularity* $(0, 0)$ *is globally asymptotically stable.*

(c) *If* $A = \frac{-3E^2+2EM+2E-M}{2E-M-1} < 1$, *then* $\operatorname{tr} DY_\nu(E, v_e) = 0$ *and the equilibrium point is a weak focus of order 1* [2].

An analogous second typo appears in the statement of Theorem 7 in [1] (page 2936) in the expressions for S. It must be replaced by the following correction:

Theorem 2. *(Theorem 7 in* [1]*) Let* $(u, v^s) \in W^s(0, 0)$ *be the stable manifold of* O *and* $(u, v^u) \in W^u(1, 0)$ *be the unstable manifold of* Q_1.

7.1 *Assuming that* $v^s > v^u$ *we obtain that:*

(a) *If* $A > \frac{2E-3E^2}{2E-1}$, *the singularity* Q_e *is a local attractor.*

(a1) *If* $S > \frac{\left(3E^2+2EA-2E-A\right)^2}{4(1-E)(A+E)}$, *the point* Q_e *is an attracting focus.*

(a2) *If* $S < \frac{\left(3E^2+2EA-2E-A\right)^2}{4(1-E)(A+E)}$, *the point* Q_e *is an attracting node.*

(b) *If* $A < \frac{2E-3E^2}{2E-1}$, *the singularity* Q_e *is a repellor.*

(b1) *If* $S > \frac{\left(-3E^2-2EA+2E+A\right)^2}{4(1-E)(A+E)}$, *then* Q_e *is an unstable focus surrounded by a stable limit cycle.*

(b2) *If* $S < \frac{\left(-3E^2-2EA+2E+A\right)^2}{4(1-E)(A+E)}$, *then* Q_e *is an unstable node and the limit cycle disappears. In this last case the singularity* $(0, 0)$ *is globally asymptotically stable.*

(c) *If* $A = \frac{2E-3E^2}{2E-1}$ *and* $S > \frac{1}{4}\frac{\left(3E^2+2EA-2E-A\right)^2}{(1-E)(A+E)}$, Q_e *is a weak focus of order 1.*

7.2 *If* $v^s < v^u$, *then the point* Q_e *is a repellor, the limit cycle disappears and the origin is globally asymptotically stable; then, a heteroclinic curve is obtained, joining* Q_e *with* $(0, 0)$.

In the appendix slight changes in some expressions must be incorporated.

In the proof of Theorem 4 (page 2940) the mistakes appear in the coefficient of y^2, in both vector fields $\bar{Z}_\eta$ and $\breve{Z}_\eta$; moreover, the expressions for A in (b) and the second Lyapunov quantity L_2 is badly written; the correct proof is:

Proof of Theorem 4. For the point Q_e, the Jacobian matrix is

$$DY_\eta(E, v_e) = \begin{pmatrix} E\left(-3E^2 - 2EA + 2EM + 2E + AM + A - M\right) & -E \\ S(1-E)(E-M)(A+E) & 0 \end{pmatrix}.$$

Hence

$$\det DY_\nu(E, v_e) = SE(1-E)(E-M)(A+E) > 0$$

and

$$\operatorname{tr} DY_\nu(E, v_e) = E\left(-3E^2 - 2EA + 2EM + 2E + AM + A - M\right),$$

and the behavior of (E, v_e) is determined by

$$T = (-2E + M + 1)A - 3E^2 + 2EM + 2E - M.$$

We have that:

(a) $\operatorname{tr} DY_\eta(E, v_e) < 0$ if and only if $A > \frac{-3E^2+2EM+2E-M}{2E-M-1}$ $(T < 0)$ and the singularity Q_e is a local attractor.

(b) $\operatorname{tr} DY_\eta(E, v_e) > 0$ if and only if $A < \frac{-3E^2+2EM+2E-M}{2E-M-1}$ and Q_e is a repellor, and by the Poincaré–Bendixson theorem at least one limit cycle surrounding the point (E, v_e) exists; the trajectories under the separatrix determined by $W^s(M, 0)$ tend to this limit cycle.

When $v^s = v^u$, the limit cycle collapses with the heteroclinic that joins the two saddle points.

(c) $\operatorname{tr} DY_\eta(E, v_e) = 0$ if and only if $A = \frac{-3E^2+2EM+2E-M}{2E-M-1} < 1$.

To determine the weakness of Q_e we employ the translation to the origin given by

$$u \to U + E \quad \text{and} \quad v \to V + v_e, \quad \text{with } v_e = \frac{(1-E)^2(E-M)^2}{2E-M-1},$$

obtaining the system

$$Z_\eta : \begin{cases} \dfrac{dU}{d\tau} = ((1-U-E)(U+E-M)(A+U+E) - (V+v_e))(U+E) \\ \dfrac{dV}{d\tau} = SU(V+v_e). \end{cases}$$

The Jordan form associated with $DZ_\eta(0, 0)$ is

$$J = \begin{pmatrix} \alpha & -H \\ H & \alpha \end{pmatrix}$$

with $\alpha = \operatorname{tr} DZ_\eta(0, 0) = 0$ and $H = \det DZ_\eta(0, 0)$, where

$$H^2 = SE\frac{(1-E)^2(E-M)^2}{2E-M-1}$$

and the matrix for the change of variables [3] is

$$N = \begin{pmatrix} Z11-\alpha & -H \\ Z21 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -H \\ \dfrac{H^2}{E} & 0 \end{pmatrix}.$$

Then the vector field Z_η becomes

$$\bar{Z}_\eta : \begin{cases} \dfrac{dx}{d\tau} = -Hy - HSxy \\ \dfrac{dy}{d\tau} = \begin{array}{l} Hx - \dfrac{H^2}{E}xy + \dfrac{H\left(1-3E+3E^2-3EM+M^2+M\right)E}{2E-M-1}y^2 \\ -H^2\dfrac{1-4E+5E^2-4EM+M^2+M}{2E-M-1}y^3 + H^3y^4. \end{array} \end{cases}$$

Carrying out a time rescaling given by $T = H\tau$, we have the canonical system

$$\check{Z}_\eta : \begin{cases} \dfrac{dx}{dT} = -y - Sxy \\ \dfrac{dy}{dT} = \begin{array}{l} x - \dfrac{H}{E}xy + \dfrac{\left(1-3E+3E^2-3EM+M^2+M\right)E}{2E-M-1}y^2 \\ -H\dfrac{1-4E+5E^2-4EM+M^2+M}{2E-M-1}y^3 + H^2y^4. \end{array} \end{cases}$$

Using the Mathematica software [4] to calculate the focal values for the vector field $\check{Z}_\eta$, the second Lyapunov quantity [2] is given by

$$L_2 = -\frac{\left(2-9E+12E^2+2M-9EM+2M^2\right)H}{8(2E-M-1)} = -\frac{H}{8(2E-M-1)}f(M, E),$$

where $L_2 < 0$, since

$$f(M, E) = 2 - 9E + 12E^2 + 2M - 9EM + 2M^2 > 0$$

for all E, such that

$$\frac{1+M}{2} < E < \frac{1}{3}\left(M + 1 + \sqrt{M^2 - M + 1}\right).$$

Thus, Q_e is a weak focus of order 1 and system (3) has a unique limit cycle. □

The unique error in the proof of Theorem 7 is in the expression for the second Lyapunov quantity L_2. The correct proof of Theorem 7 (page 2941) is:

Proof of Theorem 7. For the point Q_e, the Jacobian matrix is

$$DY_\eta(E,\ v_e) = \begin{pmatrix} -4E^3 + 3E^2(1-A) + 2AE - v_e & -E \\ Sv_e & 0 \end{pmatrix}$$

with $v_e = \frac{(1-E)^2E^2}{2E-1}$. As $v_e > 0$, then $\det DY_\eta(E,\ v_e) > 0$ and the nature of Q_e depends on

$$\operatorname{tr} DY_\eta(E,\ v_e) = -A(2E-1) + E(2-3E).$$

Q_e has the same nature as the equivalent point in system (3), that is:

If $A > \frac{2E-3E^2}{2E-1}$, the singularity Q_e is an attractor.

If $A < \frac{2E-3E^2}{2E-1}$, the singularity Q_e is a repellor surrounded by a limit cycle (the Poincaré–Bendixson theorem), when $v^s > v^u$.

If $A = \frac{2E-3E^2}{2E-1}$, the singularity Q_e is a weak focus.

Using the Mathematica software [4] we obtain that the second Lyapunov quantity [2] is $L_2 = -\frac{(2-9E+12E^2)H}{8(2E-1)}$, with $H^2 = SE\frac{(1-E)^2E^2}{2E-1}$, which is clearly negative for $E > \frac{1}{2}$. For the system (3), the uniqueness of the limit cycle, when it exists, is assured.

This limit cycle increased when the parameters changed until it intersected the heteroclinic joining Q_1 and O.

When $E \to 0$, the point Q_e is a repellor node. The heteroclinic that joined the saddle points Q_1 and O is broken (also disappearing the limit cycle); then, the origin O will be globally asymptotically stable. □

3. Conclusions

Despite the typographical mistakes in the statements and proofs of Theorems 4 and 7, the properties of system (3) (page 2933) are not altered; as a consequence, the results for system (2) (page 2932) are correct.

Therefore, the modified Rosenzweig–MacArthur model considering a new factor in the prey growth rate describing an Allee effect has interesting and varied dynamics, as was shown in [1].

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